

Radiating Kerr-Newman black hole in $f(R)$ gravity

Sushant G. Ghosh^{a, b *} and Sunil D. Maharaj^{a †}

^a *Astrophysics and Cosmology Research Unit, School of Mathematical Sciences, University of KwaZulu-Natal, Private Bag 54001, Durban 4000, South Africa and*

^b *Center for Theoretical Physics, Jamia Millia Islamia, New Delhi 110025, India*

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We derive an exact radiating (nonstationary) Kerr-Newman like black hole solutions to, constant curvature $R = R_0$ imposed, *metric* $f(R)$ gravity. This generates a geometry which precisely that of radiating Kerr-Newman-de Sitter / anti-de Sitter with $f(R)$ gravity term R_0 contributing a cosmological-like term. The structure of three horizon-like surfaces, *viz.* timelike limit, apparent horizons and event horizons, are determined. We demonstrate the existence of additional cosmological horizons, in $f(R)$ gravity model, apart from regular black hole horizons that exist in the analogous general relativity case. In particular, the known stationary Kerr-Newman black hole solutions, of $f(R)$ gravity and general relativity, are also retrieved.

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I. INTRODUCTION

The $f(R)$ gravity, where $f(R)$ is an analytic function of the Ricci scalar R , comes into existence as a straightforward extension of general relativity (GR) [1–3]. In these theories the curvature scalar R of the Lagrangian in the Einstein-Hilbert action is replaced by an arbitrary function of the curvature scalar thereby modifying GR. However, the $f(R)$ action is sufficiently general to encapsulate some of the basic characteristics of higher order gravity. It is an interesting and relatively simple alternative to GR, from the study of which yields useful conclusions [1–3], including the present day acceleration. Unlike GR, which demands metric function derivatives no higher than second order, the *metric* $f(R)$ gravity has up to fourth order derivatives [4]. This causes complications in the calculation and hence, in general, finding exact solutions in this theory is laborious. Accordingly, little is known about $f(R)$ gravity exact solutions, which deserve to be understood better. Nevertheless, recently, interesting measures have been taken to get the spherically solutions of $f(R)$ gravity [5–12]. In particular, spherically symmetric black hole (BH) solutions were obtained for a positive constant curvature scalar in [6], and a BH solution was obtained in $f(R)$ gravities by requiring the presence of a negative constant curvature scalar [7]. Also, several spherical $f(R)$ BH solutions have been obtained [7–11]. The generalization of these stationary $f(R)$ BHs to the axially symmetric case, Kerr-Newman BH, was addressed recently [14–16]. In particular, it is demonstrated [14] that the rotating BH solutions for $f(R)$ gravity can be derived starting from exact spherically symmetric solutions by a complex coordinate transformation previously developed by Newman and Janis [17] in GR. However, the axially symmetric $f(R)$ is still

unexplored, e.g., the radiating generalization of the $f(R)$ Kerr-Newman BH is still unknown. It is the purpose of this paper to generate this metrics and we also present the D-dimensional Kerr metric in $f(R)$ gravity. Thus we extend a recent work of us [18] on radiating $f(R)$ BH to include rotation. The Kerr metric [19] is undoubtedly the single most significant exact solution in the Einstein theory of GR, which represents the prototypical BH that can arise from gravitational collapse. The Kerr-Newman spacetime is associated with the exterior geometry of a rotating massive and charged BH [20].

In this paper, we obtain an exact radiating (nonstationary) Kerr-Newman-like metric to, constant scalar curvature imposed, $f(R)$ gravity in Section III. We investigate further the structure and locations of horizons of the radiating $f(R)$ Kerr-Newman metric in Section IV. We consider whether $f(R)$ gravity plays any special role in the formation of horizons in Section V. The general remarks on $f(R)$ gravity is given in Section II and we discuss briefly basic equations of HD- $f(R)$ gravity. The paper ends with concluding remarks in Section VII. We also give the HD Kerr metric, for constant curvature imposed, $f(R)$ gravity, in the appendix.

We use units which fix the speed of light and the gravitational constant via $G = c = 1$, and use the metric signature $(+, -, -, -)$.

II. BASIC EQUATIONS OF *METRIC* $f(R)$ GRAVITY

In this section we briefly review the *metric* $f(R)$ gravity in higher-dimensional (HD) spacetime. The starting point is the modified Einstein-Hilbert, D -dimensional, gravitational action [7]:

$$I = \frac{1}{16\pi} \int d^D x \sqrt{-g} (R + f(R)), \quad (1)$$

where g is the determinant of the metric g_{ab} , $(a, b = 0, 1, \dots, D-1)$, R is the scalar curvature, and $f(R)$ is the

*Electronic address: sghosh2@jmi.ac.in, sgghosh@gmail.com

†Electronic address: maharaj@ukzn.ac.za

real function defining the theory under consideration. As the simplest example, the Einstein-Hilbert action with cosmological constant Λ_f is given by $f(R) = -(D-2)\Lambda_f$.

From the above action, the equations of motion in the metric formalism are just [3]:

$$R_{ab}(1 + f'(R)) - \frac{1}{2}(R + f(R))g_{ab} + (g_{ab}\nabla^2 - \nabla_a\nabla_b)f'(R) = 2T_{ab}, \quad (2)$$

where R_{ab} is the usual Ricci tensor, the prime in $f'(R)$ denotes differentiation with respect to R , and $\nabla^2 = \nabla_a\nabla^a$ with ∇ the usual covariant derivative. Here, we are interested in obtaining the constant scalar curvature solutions $R = R_0$. Taking the trace in the Eq. (2), we get

$$2(1 + f'(R_0))R_0 - D(R_0 + f(R_0)) = 0, \quad (3)$$

where we assume that $T = T_a^a = 0$ and note that $g_a^a = \delta_a^a = D$. Eq. (3) determines the negative constant curvature scalar as [7]

$$R_0 = \frac{Df(R_0)}{2(1 + f'(R_0)) - D} \equiv D\Lambda_f. \quad (4)$$

Thus any constant curvature solution $R = R_0$ with $1 + f'(R_0) \neq 0$ obeys [3]

$$R_{ab} = \Lambda_f g_{ab} + \frac{2}{1 + f'(R_0)}T_{ab}. \quad (5)$$

For this kind of solution an effective cosmological constant may be defined as $\Lambda_f \equiv R_0/D$ for D -dimensional spacetime. In this paper we consider the case with conformal matter ($T = T_a^a = 0$). For the case of conformal matter with non-vanishing Λ_f we have again constant $R = R_0$ with $R_0 = D\Lambda_f$ and g_{ab} is a solution of $f(R)$ provided that once again $f(D\Lambda_D) = \Lambda_D(2 - D + 2f'D\Lambda_D)$. In this case the solution is dS or (A)dS depending on the sign of R_0 , just as in GR with a cosmological constant.

We note that the condition $1 + f'(R_0) > 0$ required to avoid the appearance of ghosts [1] and a necessary condition that the stationary $f(R)$ BH becomes a type of Schwarzschild-(A)dS BH. Further, we require that $f''(R) > 0$ to avoid the negative mass squared of a scalar-field degree of freedom, i.e., to avoid a tachyonic instability [1, 4]. Also $f(R)$ is a monotonic increasing function with $-1 < f'(R) < 0$.

III. RADIATING KERR-NEWMAN BLACK HOLE IN $f(R)$ THEORIES

Let us begin with the action for $f(R)$ gravity with a Maxwell term in the 4D case [11]:

$$I_g = \frac{1}{16\pi} \int d^4x \sqrt{-g} [R + f(R) - F_{ab}F^{ab}]. \quad (6)$$

The Maxwell tensor is $F_{ab} = \partial_a A_b - \partial_b A_a$, where A_a is the vector potential. From the variation of the above action

(6), the Einstein equation of motion can be written as [7, 11]

$$R_{ab} \left(1 + f'(R) \right) - \frac{1}{2} (R + f(R)) g_{ab} + (g_{ab} \nabla^2 - \nabla_a \nabla_b) f'(R) = 2T_{ab}^{EM}, \quad (7)$$

with the EMT for charged null dust [21]

$$T_{ab}^{EM} = \zeta(v, r) n_a n_b + F_{a\rho} F_b{}^\rho - \frac{g_{ab}}{4} F_{\rho\sigma} F^{\rho\sigma}. \quad (8)$$

Once again, it is easy to see that the trace of the EMT is $T^{EM} = 0$, due to the fact that $F_a^a = 0$ in 4D. However, in HD $T^{EM} \neq 0$. On the other hand, the Maxwell equations take the form $\nabla_a F^{ab} = 0$. To proceed further, with constant curvature constant R_0 , and take the trace of the Eq. (7), after some algebra, leads to

$$R_0 = \frac{2f(R_0)}{f'(R_0) - 1} \equiv 4\Lambda_f. \quad (9)$$

Thus any constant curvature solution $R = R_0$ obeys

$$R_{ab} = \Lambda_f g_{ab} + \frac{2}{1 + f'(R_0)}T_{ab}, \quad (10)$$

and an effective cosmological constant may be defined as $\Lambda_f \equiv R_0/4$.

We are looking for the exterior metric for the gravitational field produced by the charged radiating and rotating object in $f(R)$ gravity, i.e., radiating Kerr-Newman BH solutions in $f(R)$ gravity. In retarded time coordinates the metrics can be expressed as:

$$ds^2 = \frac{A}{\Sigma} [\Delta - \Theta a^2 \sin^2 \theta] dv^2 + 2\sqrt{A} [dv - a \sin^2 \theta d\phi] dr - \frac{\Sigma}{\Theta} d\theta^2 + A \frac{2a}{\Sigma} [\Delta(r^2 + a^2) - \Theta] \sin^2 \theta dv d\phi - \frac{A}{\Sigma} [\Delta(r^2 + a^2)^2 - \Theta a^2 \sin^2 \theta] \sin^2 \theta d\phi^2, \quad (11)$$

where

$$\begin{aligned} \Sigma^2 &= r^2 + a^2 \cos^2 \theta, \\ \Delta &= r^2 + a^2 - 2M(v)r + \tilde{Q}^2(v) - \frac{R_0 r^2}{12} (r^2 + a^2), \\ \Theta &= 1 + \frac{R_0}{12} a^2 \cos^2 \theta, \\ A &= \left(1 + \frac{R_0}{12} a^2 \right)^{-2}, \\ \tilde{Q}^2(v) &= \frac{2Q^2(v)}{1 + f'(R_0)}, \end{aligned}$$

and the related electromagnetic potential is

$$A_a = \frac{Q(v)r}{\Sigma^2} [\sqrt{A}, 0, 0, -\sqrt{A}a \sin^2 \theta]. \quad (12)$$

Here $M(v)$ and $Q(v)$ are functions of retarded time v identified respectively as mass and charge of spacetime,

and a is the angular momentum per unit mass. The metric (11) is a general solution of the modified Einstein field equations (10) for a charged null fluid defined by the stress energy tensor (8). It describes the exterior field of the radiating rotating charged body. Thus we have kind of charged radiating rotating metric in de Sitter/ anti-de Sitter (dS/AdS) like spacetime or radiating Kerr-Newman dS / AdS like solution and we call it radiating $f(R)$ Kerr-Newman BH. The stationary $f(R)$ Kerr-Newman BH [15, 16] in (t, r, θ, ϕ) can be obtained by means of the local coordinate transformations and replacing $M(v)$ and $Q(v)$ by constants M and Q .

The metric (11) of radiating $f(R)$ Kerr-Newman BH is a natural generalization of the stationary Kerr-Newman BH solutions of $f(R)$ gravity [15, 16], but it is Petrov type-II, whereas latter is of Petrov type-D. In addition, if $R_0 = 0$ then metric (11) makes Kerr-Newman metric. Hence, we refer solution as radiating $f(R)$ Kerr-Newman solution representing gravitational collapse of a charged null fluid in a non-flat dS/AdS like spacetime. Thus, the metric (11) bears same relation Kerr-Newman as does Vaidya metric to Schwarzschild metric. Also for $Q = 0$, the metric (11) is radiating $f(R)$ Kerr spacetime [15, 16]. If in addition $R_0 \rightarrow 0$, we have Kerr spacetime and for $a \rightarrow 0$ the metric (11) is $f(R)$ Bonnor-Vaidya spacetime which has zero angular momentum [18].

The radiating rotating charged solution discussed here are derived under the assumption of the constant curvature $R = R_0$, in which case the $f(R)$ models are equivalent to GR and a cosmological constant and also solution is time dependent.

IV. SINGULARITY AND PHYSICAL PARAMETERS OF RADIATING $f(R)$ KERR-NEWMAN BH

The metric of the radiating $f(R)$ Kerr-Newman BH solution has the form (11) with electromagnetic potential given by (12) and the energy momentum tensor (8). Here, we shall discuss the singularity structure of radiating $f(R)$ Kerr-Newman BH derived in the previous section. The easiest way to detect a singularity in a spacetime is to observe the divergence of some invariants of the Riemann tensor. We approach the singularity problem by studying the behavior of the Ricci $R = R_{ab}R^{ab}$, (R_{ab} the Ricci tensor) and Kretschmann invariants $K = R_{abcd}R^{abcd}$, (R_{abcd} the Riemann tensor). For the metric (11) they behave as:-

$$R \approx \frac{\mathcal{F}(Q(v), R_0)}{(r^2 + a^2 \cos^2 \theta)^4},$$

$$K \approx \frac{\mathcal{G}(M(v), Q(v), a, \cos \theta, R_0)}{(r^2 + a^2 \cos^2 \theta)^6}, \quad (13)$$

where \mathcal{F} and \mathcal{G} are some functions. It is sufficient to study the Kretschmann and Ricci scalars for the investigation of the spacetime curvature singularity(ies). These

invariants are regular everywhere except at the origin $r = 0$ but only at the equatorial plane $\theta = \pi/2$ for a , $M(v)$, and $Q(v) \neq 0$. Hence, the spacetime has the scalar polynomial singularity [22] at $r = 0$. The study of causal structure of the spacetime is beyond the scope of this paper and will be discussed elsewhere.

In order to further discuss the physical nature of radiating $f(R)$ Kerr-Newman BH, we introduce their kinematical parameters. Following [23–28], the null-tetrad of the metric (11) is of the form

$$l_a = [\sqrt{A}, 0, 0, -\sqrt{A}a \sin^2 \theta],$$

$$n_a = [\sqrt{A}, \frac{\Delta}{2\Sigma}, 1, 0, \sqrt{A}, \frac{\Delta}{2\Sigma}a \sin^2 \theta],$$

$$m_a = \frac{\sigma}{\sqrt{2}\rho} [\sqrt{A}ia \sin \theta, 0, \frac{\Sigma}{\Theta}, -\sqrt{A}i(r^2 + a^2) \sin \theta],$$

$$\bar{m}_a = \frac{\bar{\sigma}}{\sqrt{2}\bar{\rho}} [-\sqrt{A}ia \sin \theta, 0, \frac{\Sigma}{\Theta}, \sqrt{A}i(r^2 + a^2) \sin \theta],$$

where

$$\rho = r + ia \cos \theta$$

$$\sigma = 1 + i\sqrt{\left(\frac{R_0}{12}\right)}a \cos \theta,$$

and $\bar{\rho}$ and $\bar{\sigma}$ are complex conjugates of, respectively, ρ and σ . The null tetrad obeys null, orthogonal and metric conditions

$$l_a l^a = n_a n^a = m_a m^a = 0, \quad l_a n^a = 1,$$

$$l_a m^a = n_a \bar{m}^a = 0, \quad m_a \bar{m}^a = -1,$$

$$g_{ab} = l_a n_b + l_b n_a - m_a \bar{m}_b - m_b \bar{m}_a,$$

$$g^{ab} = l^a n^b + l^b n^a - m^a \bar{m}^b - m^b \bar{m}^a. \quad (14)$$

Following Ref. [23, 24], a null-vector decomposition of the metric is of the form

$$g_{ab} = -n_a l_b - l_a n_b + \gamma_{ab}, \quad (15)$$

where $\gamma_{ab} = m_a \bar{m}_b + m_b \bar{m}_a$. The optical behavior of null geodesics congruences is governed by the Raychaudhuri equation [24–28].

$$\frac{d\Theta}{dv} = \kappa\Theta - R_{ab}l^a l^b - \frac{1}{2}\Theta^2 - \sigma_{ab}\sigma^{ab} + \omega_{ab}\omega^{ab}, \quad (16)$$

with expansion Θ , twist ω , shear σ , and surface gravity κ . The expansion [24] of the null rays, parameterized by v , is given by

$$\Theta = \nabla_a l^a - \kappa, \quad (17)$$

where ∇ is the covariant derivative. In the present case, the surface gravity [24] is

$$\kappa = -n^a l^b \nabla_b l_a, \quad (18)$$

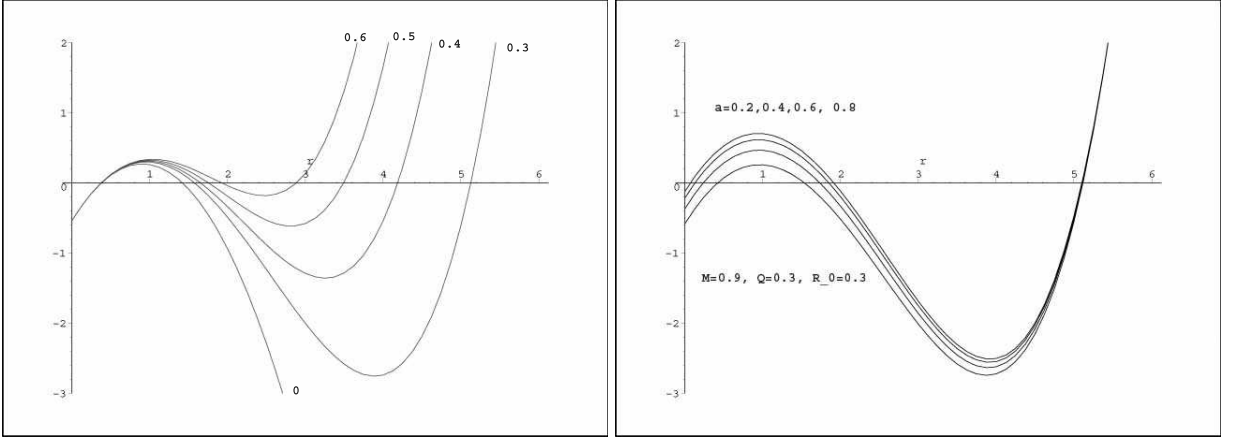


FIG. 1: A plot showing positive roots of Eq. (21). LEFT: for different values of R_0 and, $M(v) = 0.9$, $\tilde{Q}(v) = 0.3$, $\theta = 0.3$ and $a = 0.3$ which correspond to TLSs of radiating $f(R)$ Kerr-Newman BH. It is seen that for $R_0 > 0$, we have three positive roots which correspond to r_{TLS}^- (inner) and r_{TLS}^+ (outer), and r_{dTLS} (dS TLS). While for $R_0 = 0$, i.e., in GR limit, have just two outer and inner TLSs of Kerr-Newman BHs. RIGHT: for different values of a and, $M(v) = 0.9$, $\tilde{Q}(v) = 0.3$, $\theta = 0.3$ and $R_0 = 0.3$ which correspond r_{TLS}^- (inner) and r_{TLS}^+ (outer), and r_{dTLS} (dS TLS) of radiating $f(R)$ Kerr-Newman BH.

and the shear [24] takes form

$$\sigma_{ab} = \Theta_{ab} - \Theta(\gamma_c^c)\gamma_{ab}. \quad (19)$$

The luminosity due to loss of mass is given by $L_M = -dM/dv$, $L_M < 1$, and due to gauge charge by $L_Q = -dQ/dv$, where $L_M, L_Q < 1$. Both are measured in the region where d/dv is timelike [24–26].

V. THREE KINDS OF HORIZONS

A BH has three horizon-like surfaces [24]: timelike limit surface (TLS), apparent horizons (AH) and event horizons (EH). For a classical Schwarzschild BH (which does not radiate), the three surfaces EH, AH, and TLS are all identical. Upon switching on the Hawking evaporation this degeneracy is partially lifted even if the spherical symmetry stays, e.g., for Vaidya radiating BH, we have then AH=TLS, but the EH is different. If we break spherical symmetry preserving stationarity (e.g., Kerr BH), then AH=EH but $EH \neq TLS$. In general, e.g., for radiating Kerr-Newman BH, the three surfaces $AH \neq TLS \neq EH$ and they are sensitive to small perturbations.

Here we are interested in these horizons for the radiating $f(R)$ Kerr-Newman BH. As demonstrated first by York [24], the horizons may be obtained to $O(L_M, L_Q)$ by noting that (i) for a BH with small dimensionless accretion, we can define TLS's (quasi-static limit surface) as locus where $g(\partial_v, \partial_v) = g_{vv} = 0$ (ii) apparent horizons are defined as surface such that $\Theta \simeq 0$ and (iii) event horizons are surfaces such that $d\Theta/dv \simeq 0$.

A. Quasi-static limit surface or TLS

First, we calculate location of TLS surface, which for the nonstationary radiating $f(R)$ Kerr-Newman metric requires that prefactor of the dv^2 term in metric vanish; It follows from Eq. (11) that TLS will satisfy [28]

$$\Delta_r - \Delta_\theta a^2 \sin^2 \theta = 0. \quad (20)$$

This equation can be rewritten as

$$f(v, r, \theta) = \frac{R_0}{12} r^4 - \left(1 - \frac{R_0 a^2}{12}\right) r^2 + 2M(v)r + \frac{R_0}{12} \cos^2 \theta \sin^2 \theta a^4 - \cos^2 \theta a^2 - \tilde{Q}^2(v) \quad (21)$$

Equation (21) is a reduced quartic equation. It is easy to check, under condition of the discriminant in [29], Eq. (21) admits four real roots. For positive curvature $R_0 > 0$, Eq. (21), subject to restriction [29], has for all four real roots as shown in Figures (1) and (2). One of roots is negative. The other three positive roots corresponds to r_{TLS}^- (inner) and r_{TLS}^+ (outer), and r_{dTLS} (dS-like TLS). Clearly, $r_{TLS}^- < r_{TLS}^+ < r_{dTLS}$ and that r_{TLS}^- and r_{TLS}^+ are TLSs of a BH, whereas the root r_{dTLS} is supplementary TLS due to the $f(R)$ gravity term.

As mentioned above, in the limit $a \rightarrow 0$, one gets $f(R)$ Bonnor-Vaidya solution [18], and Eq. (21) takes the form

$$\frac{R_0}{12} r^4 - r^2 + 2M(v)r - \tilde{Q}^2(v) = 0. \quad (22)$$

This coincides with the nonrotational case in which case the various horizons are identified and analyzed by us in

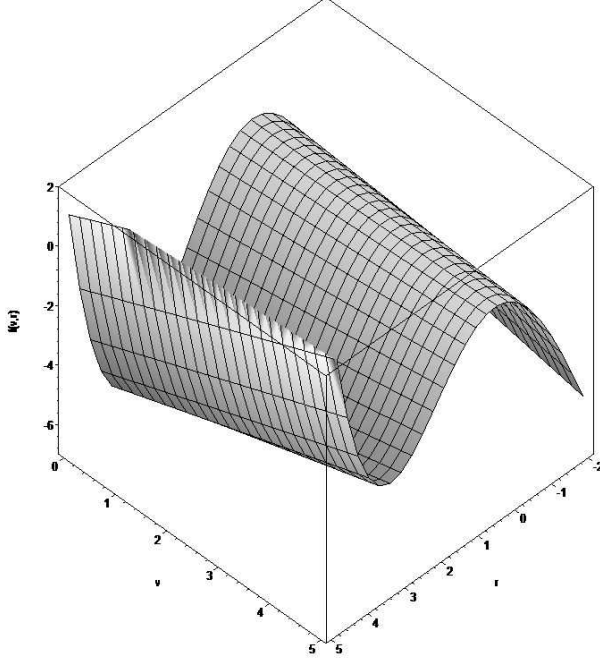


FIG. 2: A plot of the function $f(v, r, \theta)$ for $M(v) = \lambda v$ and $\tilde{Q}(v) = \mu^2 v^2$ with parameters values $\lambda = 0.04$, $\mu = 0.08$, $\theta = 0.3$, $a = 0.7$ and $R_0 = 0.3$

[18] and hence, to conserve space, we shall avoid the repetition of same. Further, in the GR limit $R_0 \rightarrow 0$; $\tilde{Q}(v) \rightarrow Q(v)$, we obtain

$$r^2 + \cos^2(\theta) a^2 - 2M(v)r + Q^2(v) = 0, \quad (23)$$

which trivially solves to

$$\begin{aligned} r_{TLS}^- &= M(v) - \sqrt{M^2(v) - a^2 \cos^2 \theta - Q^2(v)}, \\ r_{TLS}^+ &= M(v) + \sqrt{M^2(v) - a^2 \cos^2 \theta - Q^2(v)}. \end{aligned} \quad (24)$$

These are regular outer and inner TLSs for a radiating Kerr-Newman BH [20], and further in the non-rotating limit $a \rightarrow 0$, the solutions (24) reduces to

$$\begin{aligned} r_{TLS}^- &= M(v) - \sqrt{M^2(v) - Q^2(v)}, \\ r_{TLS}^+ &= M(v) + \sqrt{M^2(v) - Q^2(v)}, \end{aligned} \quad (25)$$

Again in GR limit, we get

$$r^2 - 2M(v)r + (a^2 + Q^2(v)) = 0, \quad (30)$$

which are TLS of Bonnor-Vaidya BH. Thus the radiating $f(R)$ Kerr-Newman BH, in the GR limit and $a \rightarrow 0$, degenerates to Bonnor-Vaidya BH [21].

B. Apparent Horizon

The AH is the outermost marginally trapped surface for the outgoing photons. The AH can be either null or space-like, that is, it can 'move' causally or acausally [24]. The AHs are defined as surfaces such that $\Theta \simeq 0$ [24]. Using Eqs. (14) and (18) give the expression for surface gravity

$$\kappa = \frac{1}{2\Sigma} \left[\frac{\partial \Delta_r}{\partial r} - \frac{2r}{\Sigma} \Delta_r \right], \quad (26)$$

which on inserting the expression for Δ_r , becomes

$$\begin{aligned} \kappa &= \frac{R_0}{12\Sigma^2} - \left(\frac{R_0}{6\Sigma} - \frac{1 - \frac{R_0 a^2}{12}}{\Sigma^2} \right) r^3 + \frac{2M(v)}{\Sigma^2} r^2 \\ &+ \left(\frac{1 - \frac{R_0 a^2}{12}}{\Sigma} - \frac{a^2 + Q^2(v)}{\Sigma^2} \right) r - \frac{M(v)}{\Sigma}. \end{aligned} \quad (27)$$

Eqs. (14), (17) and (26) then yield

$$\begin{aligned} \Theta &= -\frac{r}{\Sigma^2} \Delta_r = \frac{r}{\Sigma^2} \left[\frac{R_0}{12} r^4 - \left(1 - \frac{R_0 a^2}{12} \right) r^2 \right. \\ &\quad \left. + 2M(v)r - (a^2 + \tilde{Q}^2(v)) \right]. \end{aligned} \quad (28)$$

It is evident that the AHs are zeros of $\Theta = 0$. From Eq. (28), thus the AH's are given by zeros of

$$g(v, r) = \frac{R_0}{12} r^4 - \left(1 - \frac{R_0 a^2}{12} \right) r^2 + 2M(v)r - (a^2 + \tilde{Q}^2(v)) = 0. \quad (29)$$

There exist, subject to condition [29], three positive roots for $R_0 > 0$ as shown in the Figures (3) and (4). Unlike, TLS, the AH's are θ independent. Hence, unlike non-rotating BHs, they do not coincide in the rotating case. The three roots correspond to inner and outer BH AHs, and dS-like AH.

which admit solutions

$$\begin{aligned} r_{AH}^- &= M(v) - \sqrt{M^2(v) - a^2 - Q^2(v)}, \\ r_{AH}^+ &= M(v) + \sqrt{M^2(v) - a^2 - Q^2(v)}. \end{aligned} \quad (31)$$

These are regular outer and inner AHs for a radiating Kerr-Newman BH, and further in the non-rotating limit

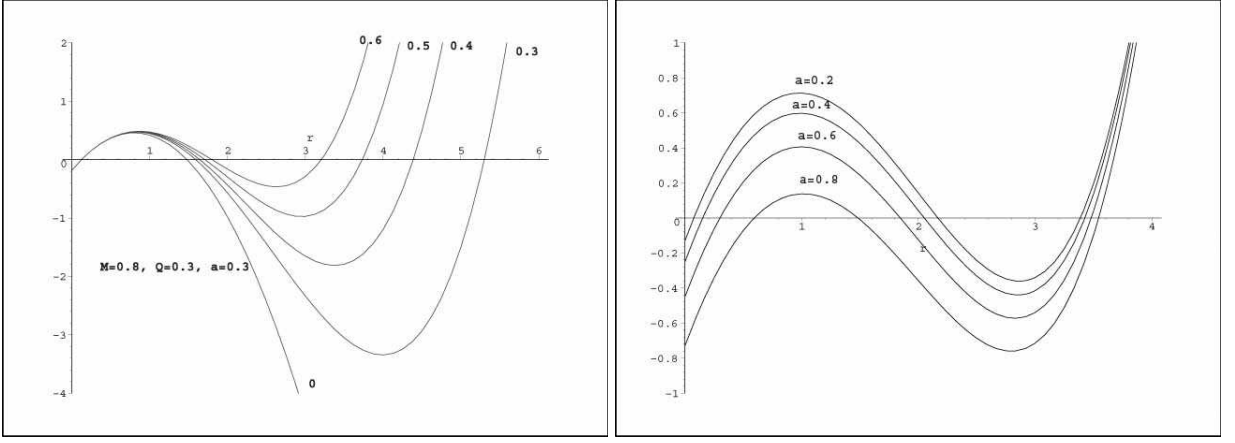


FIG. 3: A plot showing positive roots of Eq. (29). LEFT: for different values of R_0 and, $M(v) = 0.9$, $\tilde{Q}(v) = 0.3$, $\theta = 0.3$ and $a = 0.3$ which correspond to TLSs of radiating $f(R)$ Kerr-Newman BH. It is seen that for $R_0 > 0$, we have three positive roots which correspond to r_{AH}^- (inner) and r_{AH}^+ (outer), and r_{dAH} . While for $R_0 = 0$, i.e., in GR limit, have just two outer and inner AHs of Kerr-Newman BHs. RIGHT: for different values of a and, $M(v) = 0.9$, $\tilde{Q}(v) = 0.3$, $\theta = 0.3$ and $R_0 = 0.3$ which corresponds r_{AH}^- (inner) and r_{AH}^+ (outer), and r_{dAH} (dS AH) of radiating $f(R)$ Kerr-Newman BH.

$a \rightarrow 0$, the solutions (31) correspond to Bonnor-Vaidya AHs. Further, Eq. (31) in the limit $a \rightarrow 0$ becomes exactly Eq. (24). Thus AHs coincide with TLSs, for the nonrotating but radiating, Bonnor-Vaidya case [18].

The discussion in above two subsections are also valid for the stationary case discussed in Ref. [15]. In the stationary case M and Q are constant whereas in the radiating case $M(v)$ and $Q(v)$ are function of the retarded time v . Thus, Eqs. (21) and (29) are same as derived for the corresponding stationary case [15] when $M(v) = M$ and $Q(v) = Q$ with M and Q constants.

C. Event Horizon

The EH is a null three-surface which is the locus of outgoing future-directed null geodesic rays that never manage to reach arbitrarily large distances from the BH and behave such that $d\theta/dv \simeq 0$. They are determined via the Raychaudhuri Eq. (16) to $O(L_M, L_Q)$. This definition of the EH requires knowledge of the entire future of the BH. Hence, it's difficult EH exactly in non-stationary spacetime. However, York [24], gave a working definition of the EH, which is in $O(L_M, L_Q)$ equivalent to that the photons at EH unaccelerated in the sense that

$$\frac{d^2 r}{dn^2} \Big|_{r=r_{EH}} = 0, \quad (32)$$

with $d/dn = n^a \nabla_a$. This criterion enables us to distinguish the AH and the EH to necessary accuracy. It is known that [28]:

$$\frac{d^2 r}{dn^2} = \frac{1}{\sqrt{A} 2\Sigma^2} (r^2 + a^2) \frac{\partial \Delta_r}{\partial v} + \frac{\Delta_r}{2\Sigma} \kappa. \quad (33)$$

For low luminosity, the surface gravity κ can be evaluated at AH and the expression for the EH can be obtained to

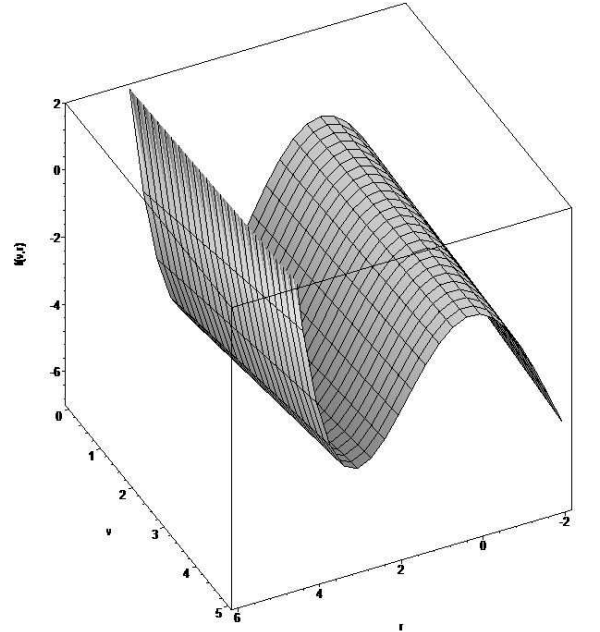


FIG. 4: A plot of the function $g(v, r)$ for $M(v) = \lambda v$ and $\tilde{Q}(v) = \mu^2 v^2$ with parameters values $\lambda = 0.04$, $\mu = 0.08$, $\theta = 0.3$ and $R_0 = 0.5$

$O(L_M, L_Q)$. Eqs. (33), (26), and the expression for Δ_r ,

lead to

$$\frac{R_0}{12}r^4 - \left(1 - \frac{R_0 a^2}{12}\right)r^2 + 2M^*(v)r - (a^2 + \tilde{Q}^{*2}(v)) = 0, \quad (34)$$

where

$$\begin{aligned} M^*(v) &= M(v) + \frac{(r^2 + a^2)}{\sqrt{A} \kappa \Sigma} L_M \\ \tilde{Q}^*(v) &= \tilde{Q}(v) + \frac{(r^2 + a^2)}{\sqrt{A} \kappa \Sigma} L_Q. \end{aligned}$$

Eq. (34) is the master equation for deciding the EHs of radiating $f(R)$ Kerr-Newman BH. It is interesting to note the mathematical similarity with its counterpart Eq. (29) for AHs. However, unlike the AHs, EHs has θ dependence as $M^*(v)$ and $Q^*(v)$ involve Σ . For stationary BH, $L_M = L_Q = 0$. Thus, unlike the stationary case [15], where AH=EH \neq TLS, we have shown that for radiating $f(R)$ Kerr-Newman BH AH \neq EH \neq TLS. Thus the expression of the EH is exactly the same as its counterpart AH given by Eq. (29) with the mass and charge replaced by the effective mass $M^*(v)$ and charge $Q^*(v)$ [25, 28]. The region between the AH and the EH is defined as a *quantum ergosphere* [24]. The GR limit will lead to the same expression as (31), with $M^*(v)$ and $Q^*(v)$ instead of, respectively, $M(v)$ and $Q(v)$.

VI. CONCLUDING REMARKS

In this paper, we have obtained a class of $f(R)$ Kerr-Newman BH in constant curvature $R = R_0$ imposed metric $f(R)$ gravity, i.e., a radiating $f(R)$ Kerr-Newman metric, describing the gravitational field exterior to radiating, rotating, charged bodies. The $f(R)$ gravity theories are designed to produce a time-varying effective cosmological constant, the BH and spherically symmetric solutions of interest are likely to represent central objects embedded in cosmological backgrounds. It is evident from analysis that the $f(R)$ gravity contributes to a cosmological-like term in the solutions and they are asymptotically dS/AdS according to the sign of R_0 , and has geometry of the Kerr-Newman dS/AdS.

The three kinds of the horizon-like surfaces of the radiating $f(R)$ Kerr-Newman BHs: TLSs, AHs, and EH are studied by the method developed by York [24] to $O(L_M, L_Q)$ by a null-vector decomposition of the metric. It turns out that for each of TLS, AH and EH, there exist three surfaces corresponding to the three positive roots r^- , r^+ and r_{dAH} . As before r^- and r^+ can be viewed, respectively, as inner and outer BH horizons, and r_{dAH} as cosmological or dS-like horizon. The fourth root r^{--} , which is negative also corresponds to cosmological horizon [20]. The analysis presented is applicable to stationary $f(R)$ Kerr-Newman BHs as well, but AHs coincide with EHs because stationary BH do not accrete, i.e., $L_M = L_Q = 0$. However, the three surfaces no more coincide with each other in radiating $f(R)$ Kerr-Newman

BHs. Thus, we have shown that the presence of the $f(R)$ gravity term R_0 produces a drastic change in the structure of these three horizons. Such a change could have a significant effect in the dynamical evolution of these horizons.

The relation between GR and any modified theory of gravity is very good way to know how much new theory is different from GR. Obviously, When $f(R) = R$, the theory reduces to GR. Since for the energy momentum tensor (8), the trace $T = 0$, consequently R , $f(R)$ and, $f'(R)$ are constant and the theory is equivalent to GR with a cosmological constant $\Lambda_f = R_0/4$. Also, the metric $f(R)$ gravity corresponds to Brans-Dicke (BD) theory with the potential term, $V(\phi) = f - Rf'(R)$, $\phi = 1 + f'(R)$ and the BD parameter $\omega_{BD} = 0$ [1, 4]. The prototype of general $f(R)$ gravity (non-constant curvature) is the model $f(R) = R - \mu^4/R$ [1, 4], where μ is a mass scale of the order of the present value of the Hubble parameter. In this model, for large values of Ricci scalar curvature (for BH solutions), the $f(R)$ function tends to $f(R) = R$, and kicks in only late as $R \rightarrow 0$; thus the gravity is not linear and modified. Thus it admits the solution derived above, but with the Λ_f modified for this choice of $f(R)$.

To conclude, It is notable that there is no exact solution in $f(R)$ gravity coupled to matter with the exception of Maxwell fields [11]. We have obtained an exact radiating rotating BH in, constant curvature, $f(R)$ gravity for charged null dust matter. The solutions presented here provides a necessary grounds to further study geometrical properties, causal structures and thermodynamics of these BH solutions, which will be subject of a future project. Further generalization of such solutions in more general $f(R)$ gravity theories is an important direction [30].

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Appendix A: Kerr Black holes in HD- $f(R)$ gravity

Lastly, we consider a D -dimensional Kerr solution in constant curvature $f(R)$ gravity, representing generalization of the exterior metric [7] for the axially symmetric rotating objects in $f(R)$ gravity. However, we shall restrict ourselves to uncharged case only because if $D > 4$, the trace of the electromagnetic EMT is not zero, which is necessary for finding the constant curvature solutions from $f(R)$ gravity.

The Kerr metric describing the stationary, axis symmetric D -dimensional spacetime in $f(R)$ gravity may by

cast in the form

$$\begin{aligned}
ds^2 = & \frac{1}{\Sigma} [\Delta - \Theta a^2 \sin^2 \theta_1] dt^2 - \frac{\Sigma}{\Delta} dr^2 - \frac{\Sigma}{\Theta} d\theta_1^2 - \frac{1}{\Sigma} [\Theta(r^2 + a^2)^2 - \Delta \sin^2 \theta_1] \sin^2 \theta_1 d\theta_2^2 \\
& - \frac{2a}{\Sigma} [\Theta(r^2 + a^2) - \Delta] \sin^2 \theta_1 dt d\theta_2^2 - r^2 \cos \theta_1 d\theta_3^2 - r^2 \cos \theta_1 \sin \theta_3^2 d\theta_4^2 - \dots \\
& - r^2 \cos \theta_1 \sin \theta_3^2 \dots \sin \theta_n^2 d\theta_{n+1}^2,
\end{aligned} \tag{A1}$$

where

$$\begin{aligned}
\Sigma &= r^2 + a^2 \cos^2 \theta_1, \\
\Delta &= (r^2 + a^2) \left[1 - \frac{2R_0 r^2}{(n+1)(n+2)(n+3)} \right] - \frac{2M}{nr^{n-2}}, \\
\Theta &= 1 + \frac{2R_0 a^2}{(n+1)(n+2)(n+3)} \cos^2 \theta_1.
\end{aligned}$$

The D -dimensional solution (A1) is a general solution of the modified Einstein field Eq. (2). The solutions are similar to D -dimensional Kerr-dS/AdS solutions. Hence, we conclude that the above rotating D -dimensional solutions of the constant curvature $f(R)$ gravity, is just

D dimensional Kerr-like solutions in dS/AdS spacetime and we call it HD- $f(R)$ Kerr solution. For $D = 4$, the metric (A1) reduces to the Kerr metric in $f(R)$ gravity [15, 16]. In addition, if $R_0 = 0$ then metric (A1) turns out to be the 4D Kerr metric. The corresponding D -dimensional radiating Kerr metric in $f(R)$ gravity can be obtained by local coordinate transformations $(t, r, \theta, \phi) \rightarrow (v, r, \theta, \phi)$ via

$$dv \rightarrow dv + \frac{r^2 + a^2}{\Delta} dr, \tag{A2}$$

$$d\phi \rightarrow d\phi + \frac{a}{\Delta} dr. \tag{A3}$$

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- [1] A. De Felice and S. Tsujikawa, *Living Rev. Relativity* **13**, 3 (2010).
 - [2] S. Nojiri and S.D. Odintsov, *Int. J. Geom. Methods Mod. Phys.* **4**, 115 (2007) 12; S. Nojiri and S.D. Odintsov, *Phys. Rep.* **505**, 59 (2011).
 - [3] S. Capozziello and M. De Laurentis, *Phys. Rept.* **509**, 167 (2011).
 - [4] M. De Laurentis, S. Capozziello: arXiv:1202.0394 [gr-qc].
 - [5] T. Multamaki and I Vilja, *Phys. Rev. D* **74**, 064022 (2007); T. Multamaki and I Vilja, *Phys. Rev. D* **76** 064021 (2007).
 - [6] G. Cognola, E. Elizalde, S. Nojiri, S.D. Odintsov and S. Zerbini, *JCAP* 0502, 010 (2005).
 - [7] de la Cruz-Dombriz, A., Dobado, A.L., Maroto *Phys. Rev. D* **80**, 124011 (2009).
 - [8] L. Sebastiani, S. Zerbini: arXiv:1012.5230 [gr-qc].
 - [9] S. E. Perez Bergliaffa and Y. E. C. de Oliveira Nunes, *Phys. Rev. D* **84**, 084006 (2011).
 - [10] A. Aghamohammadi, K. .Saaidi, M. R. Abolhasani and A. Vajdi, *Int. J. Theor. Phys.* **49**, 709 (2010).
 - [11] T. Moon, Y. S. Myung and E. J. Son, *Gen. Relativ. Grav.* **43**, 3079 (2011).
 - [12] A. M. Nzioki, S. Carloni, R. Goswami and P. K. S. Dunsby, *Phys. Rev. D* **81**, 084028 (2010).
 - [13] S. Capozziello, A. Stabile and A. Troisi, *Class. Quant. Grav.* **25**, 085004 (2008).
 - [14] S. Capozziello, M. De laurentis and A. Stabile, *Class. Quant. Grav.* **27**, 165008 (2010)
 - [15] J. A. R. Cembranos, A. de la Cruz-Dombriz and P. J. Romero, arXiv:1109.4519 [gr-qc].
 - [16] A. Larranaga, *Pramana J. Phys.* **78**, 697 (2012).
 - [17] E. T. Newman and A. I. Janis, *J. Math. Phys.* **6**, 915 (1965).
 - [18] S. G. Ghosh and S. D. Maharaj *Phys. Rev. D* **85**, 124064 (2012).
 - [19] R. P. Kerr, *Phys. Rev. Lett. D* **11**, 237 (1963).
 - [20] G. W. Gibbons and S. W. Hawking, *Phys. Rev. D* **15**, 2738 (1977).
 - [21] A. K. Dawood and S. G. Ghosh *Phys. Rev. D* **70**, 104010 (2003).
 - [22] S.W. Hawking and G.F.R. Ellis, *The Large Scale Structure of Spacetime* (Cambridge University Press, Cambridge, 1973).
 - [23] B. Carter, in *General Relativity*, edited by S. W. Hawking and I. Israel (Cambridge University Press, Cambridge, England, 1979).
 - [24] J.W. York, Jr., in *Quantum Theory of Gravity: Essays in Honor of Sixtieth Birthday of Bryce S. DeWitt*, edited by S.Christensen (Hilger,Bristol, 1984), p.135.
 - [25] R.L. Mallett, *Phys. Rev. D* **33**, 2201 (1986); B.D. Koberlein and R.L. Mallett, *Phys. Rev. D* **49**, 5111 (1994).
 - [26] B.D. Koberlein, *Phys. Rev. D* **51**, 6783 (1995).
 - [27] Xu Dian-Yan, *Class. Quantum Grav.* **15**, 153 (1998).
 - [28] Xu Dian-Yan, *Class. Quantum Grav.* **16**, 343 (1999).
 - [29] Consider a polynomial of the form $x^4 + q x^2 + r x + s = 0$, with q, r and s real and discriminant Δ , then if $\Delta > 0$ and $s < q^2/4$, then all roots are distinct and real.
 - [30] S. G. Ghosh (work in progress).